Physics I
ISI B.Math
Midterm Exam : March 1 , 2018

Total Marks: 70

Answer all questions

1.(**Marks** $= \mathbf{2} \times \mathbf{7} = \mathbf{14}$)

In this question you need to write down the correct options. There may be more than one correct option. No explanation is necessary.

(i)A point charge q is at a distance $\mathbf{r}$ from the centre of a metal sphere with a charge Q placed on it and held in a fixed position with its centre at the origin. The charge on the sphere decreases with time because it leaks into the humid atmosphere. Which of the statements about the motion of the point charge are not true ?

(a) Angular momentum about the origin is conserved
(b) Total mechanical energy is conserved
(c) $\nabla \times \mathbf{F} = 0$, where $\mathbf{F}$ is the force between the charge and the sphere.
(d) The motion remains confined to a plane.

(ii) Two particles with masses m_1 and m_2 are subject to a mutual gravitational force. S is an inertial frame S' is another inertial frame moving with velocity $\mathbf{v}$ with respect to S. For which of the following quantities will an observer in S and one is S' disagree on their measurements ?

(a) The gravitational force $\mathbf{F}$ acting on the particle of mass m_1 due to particle m_2 .
(b) The mutual potential energy $U(\mathbf{r})$ where $\mathbf{r}$ is the relative position vector between the two particles.
(c) The total mechanical energy
(d) The total angular momentum of the system about the origin.

(iii) A particle is moving in three dimensions under the influence of a potential $U(r) = \frac{1}{2}kr^2$, where k is a positive constant. Which of the statements about the motion of the particle is false ?

(a) A possible trajectory for the particle is a circular orbit.
(b) The particle can have bounded or unbounded motion depending on its total energy.
(c)The radius vector of the particle sweeps out equal areal in equal times
(d) The total energy of the particle is conserved.

(iv) A person throws a ball(at an angle of her choosing to achieve the maximum horizontal distance)with speed v from the edge of a cliff of height h. Assuming that one of these quantities is the maximum horizontal distance, which one is it ?

(a) $\frac{gh^2}{v^2}$
(b) $\sqrt{\frac{v^2h}{g}}$

(c) $\frac{v^2}{g}\sqrt{1+\frac{2gh}{v^2}}$
(d) $\frac{v^2}{g}(1+\frac{2gh}{v^2})$

(v) A cricket ball of mass m and a bowling ball of mass M (M$\gg m$) are simultaneously projected from the ground with a speed v at an angle α with the horizontal. Ignore air resistance. Which of the following statements is true ?

(a) The maximum height reached by the cricket ball is greater than that of the bowling ball.
(b) The maximum height reached by the cricket ball is equal that of the bowling ball
(c) The maximum height reached by the cricket ball is greater than that of the bowling ball
(d) It is not possible to conclude (a) , (b) or (c) on the basis of the data given
.

(vi) A particle of mass m and charge q is moving under the influence of a constant magnetic field in the z direction and a constant electric field in the x direction. Which of the following statements is true ?

(a) Total mechanical energy is conserved.
(b) Kinetic energy is conserved.
(c) Total mechanical energy is not conserved.
(d) Total linear momentum is conserved.

(vii) A mass m is attached to a spring of spring constant k . One end of the spring is attached to the wall and the mass rests on a frictionless table. The whole arrangement is immersed in a box filled with castor oil which exerts a force $-b\dot{x}$ on the block, where b is a constant. The position of the mass at its equilibrium position is $x = 0$. At $t = 0$ the mass is at $x = A$ and is released with zero velocity. If x is the position of the mass and $\dot{x}$ is its velocity and we plot x vs $\dot{x}$, the resulting curve will be in general

(a) A circle
(b) An ellipse
(c) A parabola
(d) None of the above

2. (**Marks** $= \mathbf{6+8=14}$)

a) A particle with polar coordinates r, θ which are functions of time t is moving in a plane. The velocity and acceleration of the particle can be written in plane polar coordinates as $\mathbf{v} = v_r\hat{\mathbf{r}} + v_\theta\hat{\theta}$ and $\mathbf{a} = a_r\hat{\mathbf{r}} + a_\theta\hat{\theta}$. Find $v_r, v_\theta, a_r, a_\theta$

b) An bee flies on a trajectory such that its polar coordinates at time t are given by $r = \frac{bt}{\tau^2}(2\tau - t)$; $\theta = \frac{t}{\tau}$ $(0 \leq t \leq 2\tau)$ where b and τ are positive constants. Find the velocity vector of the bee at time t and find the least speed achieved by the bee. Find the acceleration of the bee at this instant.

3.(**Marks** $= \mathbf{6+4+4=14}$)

Consider the one-dimensional potential

$$U(x) = \frac{-Wd^2(x^2+d^2)}{x^4+8d^4}$$

a) Sketch the potential and discuss the motion at various values of x. Discuss whether the motion is bounded or unbounded. Where are the equilibrium points ? Are they stable or unstable ? (Hint: You may find it convenient to rewrite the potential in terms of $y = x/d$ and $Z(y) = U/W$)

b) Find the turning points for $E = -\frac{W}{8}$ The value of W is a positive constant

c) Find the time periods of small oscillations about the stable equilibrium points.

4.(**Marks** $= \mathbf{7+7=14}$)

(a) A child of mass m sits in a swing of negligible mass suspended by a rope of length l. Assume that the dimensions of the child are negligible compared to l. His father pulls him back until the rope makes an angle 1 radian with the vertical , then pushes with a force $F = mg$ along the arc of a circle of radius l until the rope is vertical, and then releases the swing. For what duration did the father push the swing ? You may assume $sin\theta \simeq \theta$,for $\theta < 1$

(b) Now consider the swing moving through air which offers a resistive force proportional to speed such that $F_{res} = 2m\sqrt{\frac{g}{l}}(l\dot{\theta})$ where θ is the angle made by the swing with the vertical. The father pulls back the swing until the rope makes an angle 1 radian with the vertical as before, but instead of pushing with a force , releases the swing with zero speed at that point. Find θ as a function of time.

5. (**Marks** $= \mathbf{2+5+3+4=14}$)

(a) A particle of mass m and coordinates (r, θ) moves under the influence of a central force $\mathbf{F}(\mathbf{r})$. Show that angular momentum is conserved and that the orbit lies in a plane.

(b) Show that

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{L^2u^2}F(1/u)$$

where $u = \frac{1}{r}$ and L is the angular momentum of the particle. (You may find it useful to refer to question 2 a))

(c) Solve the equation for the trivial case F =0 and demonstrate that you get the expected result.

(d) If the particle is moving in a trajectory such that $r\theta =$ constant, find the potential energy of the particle.